Math 113 homework due 1/30

Go to the roots of these calculations! Group the operations. Classify them according to their complexities rather than their appearances! This, I believe, is the mission of future mathematicians

– Évariste Galois[1]

(1) Read and review sections 12.1 and 12.2 in the book and the supplemental reading.

(2) (a) if $P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 1 & 4 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 5 & 1 & 4 \end{pmatrix}$, what is QP?

(b) using P as above, find the permutations PP and PPP.

(3) (a) What permutation is the inverse of Q? What permutation that is the inverse of P?

(b) What is a general rule that lets you find the inverse of a permutation?

(c) Suppose that X is a permutation that is its own inverse, i.e. XX is the identity. What must X look like? What if you drew X as a braid?

(4) Practice Problems 12b.1 and 12b.4 from the permutations handout

(5) Draw the permutation P from question 2 as a braid. Then write it as a sequence of transpositions.

(6) Use braids to figure out what permutation of $1, 2, 3, 4$ is given by doing these transpositions: $(12)(34)(23)(12)$. Remember, the one on the right goes first

(7) (a) In class on Monday, we counted the number of permutations of $1, 2, 3, ...n$. Find a *different* way to count them, and use this to show that there are $n!$. (Here is one strategy: there are n different positions for 1 to go. After that, how many positions are left for 2? You may use this idea, or find your own totally different counting argument)

(b) What is the smallest number n such that $n!$ is larger than 1,000,000?

(8) Write your own a definition of polyhedron that allows the "picture frame" and the "little cube on a big cube" to be polyhedra, but not the "hollow box" (this will be introduced in class on Friday). What definition do you like best - yours, the definition in the book, or our definition from class? Why?

[1]Another quote from Galois which you might find (un?)inspiring: *"Genius is condemned by a malicious social organization to an eternal denial of justice in favor of fawning mediocrity!"* Though he was a brilliant mathematician and founded much of modern group theory, Galois had a tough life, a fierce temper (often aimed at his professors!) and an unfortunate death.